1. (a) Sketch, for $0 \leq x \leq 2\pi$, the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$.

(2)

(b) Write down the exact coordinates of the points where the graph meets the coordinate axes.

(3)

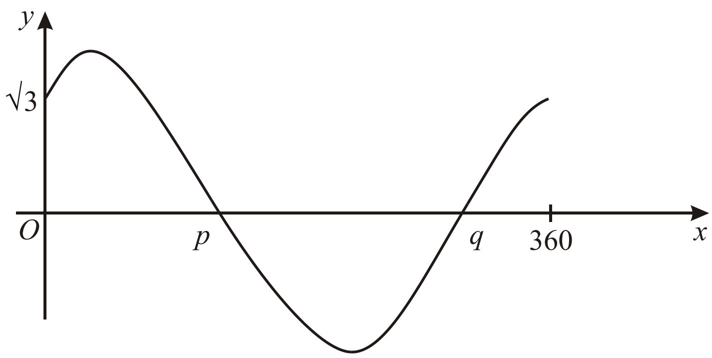
(c) Solve, for $0 \leq x \leq 2\pi$, the equation

$$\sin\left(x + \frac{\pi}{6}\right) = 0.65,$$

giving your answers in radians to 2 decimal places

(5)
(Total 10 marks)

2.



The diagram above shows the curve with equation $y = k \sin (x + 60)°$, $0 \leq x \leq 360$, where k is a constant.

The curve meets the y-axis at $(0, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

(a) Show that $k = 2$.

(1)

(b) Write down the value of p and the value of q.

(2)

The line $y = -1.6$ meets the curve at the points A and B.

(c) Find the x-coordinates of A and B, giving your answers to 1 decimal place.

(5)
(Total 8 marks)

3. The curve C has equation $y = \cos\left(x + \frac{\pi}{4}\right)$, $0 \le x \le 2\pi$.

(a) Sketch C.

(2)

(b) Write down the exact coordinates of the points at which C meets the coordinate axes.

(3)

(c) Solve, for x in the interval $0 \le x \le 2\pi$,

$$\cos\left(x + \frac{\pi}{4}\right) = 0.5,$$

giving your answers in terms of p.

(4)
(Total 9 marks)

4. (a) Sketch, for $0 \le x \le 360°$, the graph of $y = \sin(x + 30°)$.

(2)

(b) Write down the coordinates of the points at which the graph meets the axes.

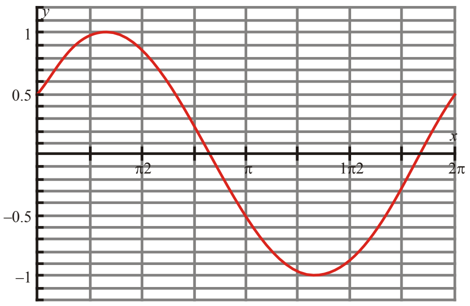
(3)

(c) Solve, for $0 \le x < 360°$, the equation

$$\sin(x + 30°) = -\tfrac{1}{2}.$$

(3)
(Total 8 marks)

1. (a)



Sine wave (anywhere with at least 2 turning points. M1

Starting on positive y-axis, going up to a max., then min. below x-axis, no further turning points in range, finishing above x-axis at $x = 2\pi$ or 360°. There must be <u>some</u> indication of scale on the y-axis... (e.g. 1, –1 or 0.5) A1 2

Ignore parts of the graph outside 0 to 2π.

n.b. Give credit if necessary for what is seen on an initial sketch (before any transformation has been performed)

(b) $\left(0, \frac{1}{2}\right), \left(\frac{5\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$

(Ignore any extra solutions) (Not 150°, 330°) B1, B1, B1 3

$\left(\pi - \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are sufficient, but if <u>both</u> are seen allow B1B0.

The zeros are not required, i.e. allow 0.5, etc. (and also coordinates the wrong way round).

These marks are also awarded if the exact intercept values are seen in part (a), but if values in (b) and (a) are contradictory, (b) takes precedence.

(c) awrt 0.71 radians (0.70758...), or awrt 40.5° (40.5416...) (α) B1

$(\pi - \alpha)$ (2.43...) or $(180 - \alpha)$ <u>if α is in degrees</u>. $\left[\underline{\text{NOT}}\ \pi - \left(\alpha - \frac{\pi}{6}\right)\right]$ M1

Subtract $\frac{\pi}{6}$ from α (or from $(\pi - \alpha)$)... or subtract 30 <u>if α is in degrees</u> M1

0.18 (or 0.06π), 1.91 (or 0.61π) Allow awrt A1, A1 5

(The 1st A mark is dependent on just the 2nd M mark)

B1: if the required values of α is <u>not seen</u>, this mark can be given by implication if a final answer rounding to 0.18 or 0.19 (or a final answer rounding to 1.91 or 1.90) is achieved. (Also see premature approx. note*).

<u>Special case:</u>

$\sin\left(x + \frac{\pi}{6}\right) = 0.65 \Rightarrow \sin x + \sin\frac{\pi}{6} = 0.65 \Rightarrow \sin x = 0.15$

$x = \arcsin 0.15 = 0.15056...$ and $x = \pi - 0.15056 = 2.99$ (B0M1M0A0A0)

(This special case mark is also available for degrees... 180 – 8.62...)

Extra solutions outside 0 to 2π: Ignore.

Extra solutions between 0 and 2π: Loses the final A mark.

*<u>Premature approximation</u>:

e.g. $\alpha = 41°$, 180 – 41 =139, 41 – 30 = 11 and 139 – 30 = 109

Changing to radians: 0.19 and 1.90

This would score B1 (required value of α not seen, but there is a final answer 0.19 (or 1.90)), M1M1A0A0.

[10]

2. (a) Substitute $x = 0$, $y = \sqrt{3}$ to give $\sqrt{3} = k\frac{\sqrt{3}}{2} \Rightarrow k = 2$

(or verify result) must see $\frac{\sqrt{3}}{2}$ B1 1

(b) $p = 120$, $q = 300$ (f.t. on $p + 180$) B1, B1ft 2

(c) *arc* sin(–0.8) = –53.1 or *arc* sin(0.8) = 53.1 B1

$(x + 60) = 180 - \arcsin(-0.8)$ or equivalent $180 + \arcsin 0.8$ M1

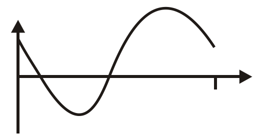
First value of $x = 233.1 - 60$, i.e. $x = 173.1$ A1

OR $(x + 60) = 360 + \arcsin(-0.8)$ or equivalent $360 - \arcsin 0.8$,

i.e. $x = 246.9$ M1, A1 5

[8]

3. (a)



Cosine/Sine shape, period 2π B1

All "features" correct. B1 2

For first B1, if full domain is not shown, at least 0 to π must be shown, and in this case there must be indication of scale on the x-axis.
For second B1, ignore anything outside the 0 to 2π domain.

(b) $\left(0, \frac{1}{\sqrt{2}}\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right)$ or, e.g. $x = 0, y = \frac{1}{\sqrt{2}}$. B1 B1 B1 3

Allow 45° and 225°, allow any exact form.
Penalties: Non-exact, lose 1 mark (maximum).
Missing zeros, or coordinates wrong way round, lose 1 mark (maximum).

(c) $\left(x + \frac{\pi}{4} =\right) \frac{\pi}{3}$ or 60° or 1.05 (2 d.p. at least) B1

Other value $\left(2\pi - \frac{\pi}{3} =\right) \frac{5\pi}{3}$ M1

Subtract $\frac{\pi}{4}$ M1

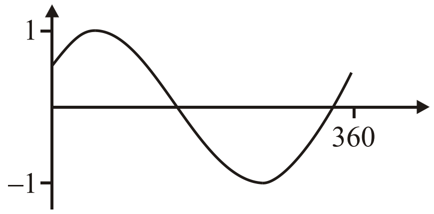
(For the M marks, allow working in degrees or radians to 2 d.p. at least, but not degrees and radians mixed unless corrected later.)

$x = \frac{\pi}{12}, x = \frac{17\pi}{12}$ A1 4

Allow 0.08π and 1.42π (2 d.p. or better)
Ignore extras outside 0 to 2π.
n.b. There are other correct approaches for the first M1, using the symmetry of the graph.

[9]

4. (a)



Scales (–1, 1 and 360)	B1
Shape, position	B1

(b)	(0, 0.5) (150, 0) (330, 0)	B1 B1 B1
(c)	(x + 30=) 210° or 330°	B1
	One of these	
	x = 180° , 300°	
	M: Subtract 30, A: Both	M1 A1

[8]

1. Sketches of the graph of $y = \sin\left(x + \frac{\pi}{6}\right)$ in part (a) were generally disappointing. Although most candidates were awarded a generous method mark for the shape of their graph, many lost the accuracy mark, which required a good sketch for the full domain with features such as turning points, scale and intersections with the axes 'in the right place'. In part (b), the exact coordinates of the points of intersection with the axes were required. Many candidates were clearly uncomfortable working in radians and lost marks through giving their x values in degrees, and those who did use radians sometimes gave rounded decimals instead of exact values. The intersection point (0, 0.5) was often omitted.

Part (c) solutions varied considerably in standard from the fully correct to those that began with $\sin\left(x + \frac{\pi}{6}\right) = \sin x + \sin\frac{\pi}{6} = 0.65$. The most common mistakes were: failing to include the 'second solution', subtracting from π after subtracting $\frac{\pi}{6}$, leaving answers in degrees instead of radians, mixing degrees and radians, and approximating prematurely so that the final answers were insufficiently accurate.

2. Not enough steps of working were shown by many candidates here. Frequently k=√3/sin60=2 was stated without reference to the fact that sin60=√3/2

(b) This was well answered and many candidates had both parts correct. Of those who didn't quite a few gained a B1 follow through for p+180.

(c) Most candidates gained a few marks here, but not many had full marks. The most common mistakes were to find -53.1, then subtract the 60 to get -113.1 resulting in 180-113.1 and 360-113.1. Some didn't give their answers to 1dp, and others mistakenly stated that 0.8=sinx+sin60 and proceeded to geta range of erroneous solutions!

3. In part (a), there were many good attempts to sketch the cosine curve, with the translation "to the left" well understood. One of the two available marks was often lost, however, sometimes because curves were incomplete for the required domain and sometimes because the position of the maximum was incorrect.

<u>Exact</u> coordinates were demanded in part (b), and few candidates scored all 3 marks available here, even though the mark scheme generously allowed *x* values in degrees as an alternative to radians. The point $\left(0, \frac{1}{\sqrt{2}}\right)$ was usually omitted or given as a rounded decimal, zero coordinates were often omitted and coordinates were sometimes reversed.

While a few candidates used their sketch to help them to solve the equation in part (c), most tackled this independently, with variable success. The first correct solution $\frac{\pi}{12}$ was often seen, the other $\left(\frac{17\pi}{12}\right)$ much less frequently. A common mistake was to calculate $2\pi - \left(\frac{\pi}{4} - \frac{\pi}{3}\right)$ instead of $\left(2\pi - \frac{\pi}{3}\right) - \frac{\pi}{4}$. Working in degrees and changing to radians at the end was a successful strategy for many candidates, but sometimes degrees and radians were combined to give confused values.

4. Some good graph sketches were seen in part (a), where the usual method was either direct "plotting" or translation of the graph of $y = \sin x$. Many candidates, however, did lose a mark here because their graph was incomplete, stopping at the x-axis where $x = 330°$, or because indication of scale on the axes was inadequate. A few translated the graph of $y = \sin x$ in the wrong direction, but were given follow-through credit in the remainder of the question where possible.

Those who produced good sketches were usually able to give the correct coordinates for the intersection points with the x-axis, but the intersection (0, 0.5) with the y-axis was often carelessly omitted.

While a few candidates were able to make efficient use of their sketches to solve the equation in part (c), the majority followed the usual equation-solving procedure and often had difficulty in reaching the required solutions. Typically, $(x + 30) = -30$ was seen, leading to $x = -60$, but complete methods showing clearly $(x + 30) = 210$ and $(x + 30) = 330$ were disappointingly rare.